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LETTER TO THE EDITOR

Fallacies in the understanding of the quenching of the Hall effect: I

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Abstract. A rigorous classical calculation of the Hall effect is done for quasi-one-dimensional systems in weak magnetic fields. The final outcome dispels the notion that the observed quenching of the Hall effect in such systems could arise because of the electrons impinging frequently against the two edges of the system. The condition for the quenching is derived but it turns out to be too strict to be feasible.

A curious new experimental result, commonly known as 'quenching of the Hall effect' [1, 2], has revealed that the Hall voltage in quasi-one-dimensional (1D) systems does not build up as soon as the magnetic field, B_z, is switched on (in the usual geometry, with the system being in the xy plane, the current in the x direction and B in the normal, z direction) at low temperatures ($T \simeq 0$ K); it begins to develop only for $B_z > B_z^{\mathrm{crit}}$, where B_z^{crit} is of the order of 0.1 T. Two attempts have been made to explain this phenomenon. Beenakker and van Houten [3], on one hand, have proposed a semi-classical explanation which in simple terms suggests that so long as the edge states are suppressed because of the electrons on their traversing trajectories colliding randomly against the upper and the lower edges, the Hall voltage will not be developed. On the other hand, the author [4] has put forward a quantum mechanical argument that is based on a subtle Josephson-type effect predicted (previously, [5]) to be present in *narrow* Hall samples.

In order to ascertain if a classical/semi-classical explanation of the effect in question would be constrained by any severe requirement, and if it does, then whether such a requirement could possibly be fulfilled, we first do a very systematic classical analysis following Pippard [6] for the conditions pertinent to the above experiments, and then, in a future publication, we will present a semi-classical analysis based on the ideas of 'skipping orbits' given in [7]. Both the analyses indicate that the normal Hall effect should be observed under the conditions mentioned above in case one confines oneself to the classical or semi-classical framework. Thus it is necessary that one must go beyond it, to understand the phenomenon of quenching of the Hall effect, as has been done by the author in [4].

We are concerned with the low-field behaviour, when B_z is so small that the smallest cyclotron orbit diameter is greater than the width of the quasi-1D system. Then every trajectory starts and finishes on one edge or the other. We define the parameter $\beta = w/R$ where w is the width of the system and R is the radius of the cyclotron orbit; for narrow systems at small B_z we have $\beta \ll 1$. In figure 1 two types of trajectory are shown reaching the lower edge at the origin of coordinates 0; the grazing orbit that separates

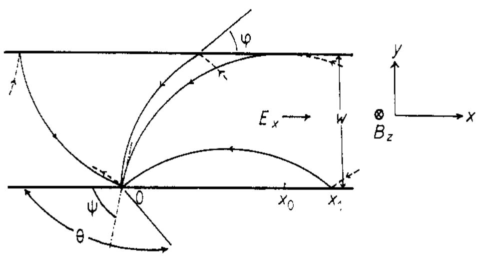

Figure 1. The experimental arrangement of the quasi-1D sample and examples of different trajectories running along an edge as well as between the edges. ψ is the angle corresponding to the grazing orbit.

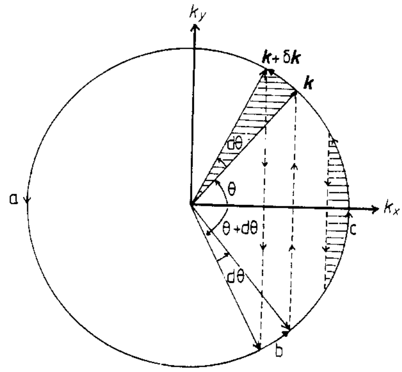

Figure 2. Orbits of a free electron in momentum space: (*a*) total orbit; (*b*) orbits truncated because of reflections at the edges; and (*c*) skipping orbits along an edge. The shaded area indicates the portion traversed by the k-vector as an electron goes from one edge to the other.

them is also shown. We shall be concerned only with the electrons as they reach and leave the edges, where the potential is $\pm V_0 - E_x x$ (where the $2V_0 \equiv$ Hall voltage, and E_x is the electric field in the x direction in the plane of the system). We are interested in obtaining the condition under which V_0 will vanish.

It is easy to check that

$$\beta = \cos\varphi - \cos\theta \tag{1}$$

and that the orbit that spans the entire width leaves the upper edge at x_0 where

$$x_0 = R(\sin\theta - \sin\varphi). \tag{2}$$

If the orbit both starts and finishes on the lower edge, it starts at x_1, where

$$x_1 = 2R\sin\theta. \tag{3}$$

Anywhere in the quasi-1D system the departure from equilibrium of the electron distribution is specified by the displacement, measured as an energy ε, of the Fermi level at each point on the Fermi surface. The ε, averaged over the Fermi surface, must vanish to ensure the space-charge neutrality.

Suppose there are no internal collisions† and that the electrons leaving the upper edge have energy $-\varepsilon'$ and those leaving the lower edge have energy ε'; ε' is to be found by requiring that the normal current, J_y in the present geometry, shall vanish. For $0 < \theta < \psi$ (ψ being the value of θ for the grazing orbit in figure 1), electrons leave the lower edge with kinetic energy ε' and regain it with $\varepsilon' - eE_x x_1$, having suffered a potential energy increase of $eE_x x_1$. If $\psi < \theta < \pi$, they come from the upper edge and arrive at the lower edge with kinetic energy $-\varepsilon' - 2eV_0 - eE_x x_0$. If we include the possibility of *specular reflections* (see, for example, [8]) from the edges and introduce p as the probability that an electron after hitting an edge is reflected *specularly* then the expressions of kinetic energy are modified as follows.

Consider first the situation where $0 < \theta < \beta$. From an arbitrary point on the lower edge a fraction p of electrons is specularly reflected and suppose the fraction $(1-p)$ goes a distance x_1 along the same edge. Then $p(1-p)$ will reach a distance $2x_1$ after the

† This is a valid assumption in the present context where the electron movement is believed to be ballistic [2, 3].

specular reflection at x_1, $p^2(1-p)$ will go $3x_1$, and so on. Thus the average distance an electron can reach in this manner will be

$$\bar{x}_1 = x_1(1-p)(1+2p+3p^2+\ldots) = x_1/(1-p) \tag{4}$$

i.e. it increases by a factor $(1-p)^{-1}$ owing to the specular reflections. So we get

$$\varepsilon(\theta) = \varepsilon' - 2eE_xR\sin\theta/(1-p) \qquad \text{for } 0<\theta<\psi. \tag{5}$$

Further, in the case $\psi<\theta<\pi$ where the electrons are scattered from both the edges alternately, suppose $(1-p)$ come from top at x_0 onto the bottom edge. These come with the kinetic energy $-\varepsilon' - 2eV_0 - eE_xx_0$. On specular reflection from the bottom edge at $2x_0$, $p(1-p)$ go up to the top edge with kinetic energy $\varepsilon' - 2eE_xx_0$. Thus $p^2(1-p)$, $p^4(1-p), \ldots$ are reflected specularly from the top at $3x_0, 5x_0, \ldots$ respectively and $p^3(1-p)$, $p^5(1-p), \ldots$ are reflected from the bottom at $4x_0, 6x_0, \ldots$ respectively. Adding the kinetic energies contributed by top and the bottom edges, we get

$$\begin{aligned}\varepsilon(\theta) &= (1-p)[(-\varepsilon' + p\varepsilon' - p^2\varepsilon' + p^3\varepsilon' - \ldots) - 2eV_0(1+p^2+p^4+\ldots) \\ &\qquad - eE_xx_0(1+2p+3p^2+\ldots)] \\ &= -\varepsilon'(1-p)/(1+p) - 2eV_0/(1+p) - eE_xR(\sin\theta - \sin\varphi)/(1-p).\end{aligned} \tag{6}$$

For the quasi-1D channel of interest here,

$$\beta = w/R = \cos\varphi - \cos\theta \simeq 0$$

so that $\varphi \simeq \theta + 2n\pi$, i.e. $\sin\theta \simeq \sin\varphi$. Also, for $\theta<\psi$, $\cos\psi = 1-\beta \simeq 1$ (for $\beta \simeq 0$), i.e. $\psi \simeq 0$, and the smaller ψ is, the closer p will be to 1 for small angles of incidence $(\theta<\psi)$. Thus, as $\sin\theta \to 0$, $(1-p)\to 0$. Consequently, for the system of interest,

$$\varepsilon(\theta) = \begin{cases} \varepsilon' - 2eE_xR & 0<\theta<\psi \\ -[(1-p)/(1+p)]\varepsilon' - 2eV_0/(1+p) & \psi<\theta<\pi. \end{cases} \tag{7}$$

Note that p may not be close to unity for $\psi<\theta<\pi$ when the trajectories run between the edges.

In order to determine ε' and E_x in terms of V_0 we require that there shall be charge neutrality at the edges and that J_y shall also vanish at the edges. If the latter is achieved J_y automatically vanishes everywhere, but internal charge neutrality is not automatic. However, the local value of V adjusts itself everywhere to shrink or expand the Fermi surface uniformly until neutrality is reached.

For the charge neutrality at the edges we impose

$$\int_0^\pi \varepsilon\,\mathrm{d}\theta = 0 \tag{8}$$

which, for (7), gives

$$\begin{aligned}\varepsilon' &= \{2eE_xR\psi + [2eV_0/(1+p)](\pi-\psi)\}/\{\psi - [(1-p)/(1+p)](\pi-\psi)\} \\ &\simeq -2eV_0/(1-p) \qquad \text{for } \psi \simeq 0.\end{aligned} \tag{9}$$

To make J_y vanish we must have

$$\int_0^\pi \varepsilon\sin\theta\,\mathrm{d}\theta = 0$$

and for (7) this gives

$$(\varepsilon' - 2eE_xR)\beta = \{[(1-p)\varepsilon' + 2eV_0]/(1+p)\}(\beta - 2) \tag{10}$$

or, for $\beta \simeq 0$,

$$\varepsilon' \simeq -2eV_0/(1-p). \tag{9'}$$

Finally we determine the total current to obtain the resistance. Associated with the displacement of the Fermi surface by $\varepsilon(\theta)$ there is an excess number of carriers $n\varepsilon\,\delta\theta/(2\pi E_F)$ due to the elements $\delta\theta$ each having momentum $p_y = mv_F \sin\theta$ and moving with velocity $v_F \sin\theta$ in the y direction (here n is the number of electrons per unit area) [6]. These electrons convect in a positive direction to increase the momentum at the rate

$$\dot{P}_y = \int_0^\pi (n\varepsilon\,\mathrm{d}\theta/2\pi E_F) m v_F^2 \sin^2\theta = (n/\pi)\int_0^\pi \varepsilon \sin^2\theta\,\mathrm{d}\theta. \tag{11}$$

At the same time the Hall field increases P_y by $2neV_0$ and the Lorentz force increases it by B_zI_x, where I_x is the total current. In the steady state, the net change must vanish, i.e.

$$B_zI_x + 2neV_0 + (2n/\pi)\int_0^\pi \varepsilon\sin^2\theta\,\mathrm{d}\theta = 0. \tag{12}$$

This gives for $\varepsilon(\theta)$ as in (7),

$$B_zI_x + 2neV_0 - n[(1-p)\varepsilon' + 2eV_0]/(1+p) = 0$$

or

$$I_x = -2neV_0/B_z \tag{13}$$

using $\varepsilon' = -2eV_0/(1-p)$ from (9) and (9′). Thus in (13) we obtain the *normal* Hall effect in our quasi-1D system for small B_z. We therefore concluded that the simple and straightforward classical considerations allowing electrons to impinge freely against the two closely placed edges of a quasi-1D system yield nothing spectacular over and above the normal Hall effect even if we allow the reflections from the edges to be specular with *non-zero* probability. However, in the limiting situation of $p = 1$, the V_0 will vanish according to (9). That is, if the reflections from the two edges are specular with *probability one* for *any* angle of incidence, then the Hall voltage, V_0, can vanish and the quenching of the Hall effect can happen. We feel this condition is too stringent to be fulfilled in an actual experiment. An infinitesimal deviation in value of p from 1 on one of the collisions with an edge will build up on the subsequent collisions and will result in the development of a Hall voltage. This will happen with much ease with almost all the trajectories except those that are incident on an edge at very small angles ($\theta \sim 0$).

The above arguments can alternatively be summarised as follows. The movement of a free electron can be described in $\boldsymbol{k}$-space by the movement of the radius vector $\boldsymbol{k}$ about the origin (see figure 2). In a narrow system like ours $\boldsymbol{k}$ traces only small segments of the circle such as those shown by the shaded portions. Suppose $\boldsymbol{k}$ changes from an arbitrary value $\boldsymbol{k}$ to $\boldsymbol{k} + \delta\boldsymbol{k}$ as the electron moves from one edge to the other and then a *specular* reflection occurs that keeps k_x unchanged both in direction and magnitude and reverses the direction of k_y but leaves its magnitude unchanged. Now the k-vector changes from $\boldsymbol{k} + \delta\boldsymbol{k}$ to $\boldsymbol{k}$ and the electron moves back to the first edge and again encounters a *specular* reflection. If this corresponds to the change in angle (between $\boldsymbol{k}$ and the x axis) from θ to $\theta + \mathrm{d}\theta$, and vice versa, then

$$\delta v_y = \pm v(\sin(\theta + \mathrm{d}\theta) - \sin\theta) \simeq \pm\, v\cos\theta\,\mathrm{d}\theta \tag{14}$$

taking $\sin(\mathrm{d}\,\theta) \simeq \mathrm{d}\,\theta$ and $\cos(\mathrm{d}\,\theta) \simeq 1$ for very small $\mathrm{d}\,\theta$. As long as *all* the trajectories, irrespective of their angle of incidence on an edge, undergo specular reflections after *each* collision with the edges, i.e. *all* reflections are specular with probability 1, then the positive and negative values of δv_y will be exactly equal individually for each trajectory and the net transfer of charge from one edge to the other will be zero. It is clear that this condition is too ideal to be strictly obeyed in an experiment.

In conclusion, we show that the quenching of the Hall effect in quasi-1D systems cannot be understood to be occurring merely because of frequent impinging of the electrons against the two edges of the system. Within this classical framework the Hall effect can be quenched under the extreme ideal condition where all the collisions of the electrons with the edges of the system result in *specular* reflections with *probability* 1.

I am grateful to Professor Sir A B Pippard for numerous discussions and for allowing access to the manuscript of his book on magnetoresistance ([6], unpublished at that time) which forms the basis for the present calculation. Thanks are due to Professor Sir S F Edwards for hospitality at the Cavendish Laboratory where this work was done, and to Professors M Pepper and V Srinivasan for discussions. Arguments based on figure 2 resulted out of discussions with Professor Pepper.

References

[1] Roukes M L, Sherer A, Allen S J Jr, Craighead H G, Ruthin R M, Beebe E D and Harbison J P 1987 *Phys. Rev. Lett.* **59** 3011
[2] Ford C J B, Thornton T J, Newbury R, Pepper M, Ahmed H, Peacock D C, Ritchie D A, Frost J E F and Jones G A C 1988 *Phys. Rev.* B **38** 8518
[3] Beenakker C W J and von Houten H 1988 *Phys. Rev. Lett.* **60** 2406
[4] Srivastava V 1989 *J. Phys.: Condens. Matter* **1** at press
[5] Srivastava V 1988 *J. Phys. C: Solid State Phys.* **21** L815
[6] Pippard A B 1988 *Magnetoresistance in Metals, Cambridge Studies in Low-Temperature Physics* vol 2 (Cambridge: CUP)
[7] Lifshitz I M and Kosevich A M 1953 *Dokl. Akad. Nauk SSSR* **91** 795
[8] Khaikin M S 1969 *Adv. Phys.* **18** 1